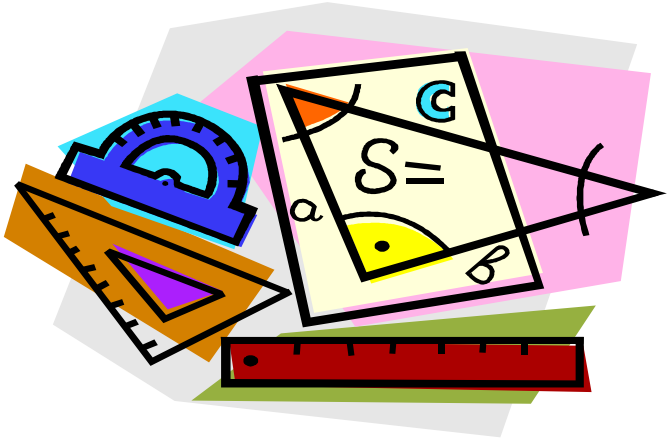
# Level III




## Do NOT open until you are told to do so.

March 23, 2023

1. A meteorologist predicts a 40% chance of rain each day for the next four days. If the meteorologist is perfectly accurate, what is the probability to the nearest percent that there will be rain on at least one day of the four days?

   a. 85% b. 87% c. 60% d. 75% e. 50%

2. Ted agrees to play a trivia game where a player receives 5 points for answering an easy question and 11 points for answering a hard question. What is the largest integer that cannot be Ted's score?

   a. 34 b. 38 c. 39 d. 43 e. 47

3. If $a^x = c^q$ and $c^y = a^z$, then which of the following equations is true?

   a. $xy = qz$ b. $\frac{x}{y} = \frac{q}{z}$ c. $x + y = q + z$ d. $x^y = q^z$ e. none of these

4. How many ounces of water are required to reduce 9 ounces of shaving lotion containing 50% alcohol to a lotion containing 30% alcohol?

   a. 4.5 b. 5 c. 8 d. 6 e. 3

5. A line with slope 6 intersects the curve $y = x^2$ at two distinct points. If the $y$-coordinate of one point is 4 times the $y$-coordinate of the other point and the two points do not lie in the same quadrant, then what is the sum of the $y$-coordinates?

   a. 20 b. 180 c. 125 d. 80 e. 245

6. Let $S=\left\{(x,y)\middle| x \text{ and } y \text{ are integers and solutions of } y=\dfrac{x}{x-2}\right\}$. What is the sum of all the $y$-coordinates of the set $S$?

   a. 0 b. 4 c. 2 d. 7 e. The sum is infinite.

7. Let $f(x)=|5x+3|-|3x-5|$. What is the minimum value of $f(x)$?

   a. $-\dfrac{34}{5}$ b. $\dfrac{40}{3}$ c. $0$ d. $\dfrac{34}{5}$ e. $-\dfrac{40}{3}$

8. Let $a$ and $b$ represent the legs of a right triangle, and let $c$ represent the hypotenuse. If $a+b=\sqrt{65}$ and $c=\sqrt{33}$, what is the area of the triangle?

   a. $4\sqrt{3}$ b. $5$ c. $10$ d. $8$ e. $16$

9. If the discriminant of $ax^2+2bx+c=0$ is zero for real numbers $a$, $b$, and $c$, then which of the following is always a true statement about $a$, $b$, and $c$?

   a. They are all negative numbers.
   b. They are all positive numbers.
   c. Only $b$ is negative and $a$ and $c$ are positive.
   d. They are consecutive terms of an arithmetic sequence.
   e. They are consecutive terms of a geometric sequence.

10. If you are given $\log(8)=0.9031$ and $\log(9)=0.9542$, then which of the following could not be approximated to at least three decimal places using only paper and pencil.

    a. $\log(1.25)$ b. $\log(15)$ c. $\log(17)$ d. $\log(600)$ e. $\log(72)$

11. Let $f(x,y)=\frac{x}{|x|}+\frac{y}{|y|}+\frac{xy}{|xy|}$. What is the range of *f*?

a. $(-\infty,\infty)$ b. $[1,\infty)$ c. $\{-3,-1,1,3\}$ d. $\{-1,1,3\}$ e. $\{-1,3\}$

12. What is the domain of the real-valued function $f(x)=\sqrt{5-\sqrt{(9-x)}}$ ?

a. $[-\infty,9]$ b. $[-5,9]$ c. $[-9,9]$ d. $[-16,9]$ e. $[5,9]$

13. If the graphs of the functions $f(x)=b(x-m)^2+n$ and $g(x)=x-m$ intersect, then what is the greatest possible value of the product of $b$ and $n$?

a. $\frac{1}{4}$ b. 1 c. $\frac{3}{4}$ d. $\frac{1}{2}$ e. 2

14. What is the sum of all zeroes (both real and complex) of the function $f(x)=\frac{x^3-0.125}{x-0.5}$?

a. $-0.5+0.5i\sqrt{3}$ b. $-0.5$ c. $0.5$ d. $-0.5-0.5i\sqrt{3}$ e. $0$

15. If $k=\log_a b$, then which of the following is equal to $\log_a\left(\sqrt[3]{b^2}\right)-\log_b\left(a^{-3}\right)$?

a. $\frac{2k^2-9}{3k}$ b. $\frac{2k}{3}$ c. $\frac{2k^2+9}{3k}$ d. $\frac{k^2-9}{3k}$ e. $\frac{k^2+9}{3k}$

16. As a bonus for signing up with U-Tunes, you get to choose any four of this week's top fifteen songs to download free. In how many ways can you make this selection?

a. 1155 b. 1195 c. 1395 d. 1375 e. none of these

17. Two congruent parallel chords are 8 inches apart in a circle of radius 8 inches. What is the area in square inches of the region enclosed by these two chords and the circle?

a. $\frac{64\pi}{3}-32\sqrt{3}$ b. $32\sqrt{3}+\frac{128\pi}{3}$ c. $16\sqrt{3}+\frac{64\pi}{3}$ d. $16\sqrt{3}+\frac{128\pi}{3}$ e. $32\sqrt{3}+\frac{64\pi}{3}$

18. What is the maximum value of the function $P=6x+5y$ subject to the constraints: $x\geq 0$; $y\geq 0$; $x+y\leq 76$; and $x-3y\geq 0$?

a. 456 b. 437 c. 380 d. 519 e. 446

19. There are several pairs of integers $(a,b)$ satisfying $a^2-4a+b^2-8b=30$. What is the sum of the sum of the coordinates of all such points?

a. 60 b. 74 c. 50 d. 72 e. 0

20. An elementary school teacher had her purse stolen. The thief had to be David, Theo, Judy, Lillian, or Margaret. When questioned, each child made three statements.

    David: (1) I did not take the purse. (2) I did not know Margaret before I enrolled in this school. (3) Theo did it.
    Theo: (1) I am not guilty. (2) Margaret did it. (3) Lillian is lying when she says I stole the purse.
    Judy: (1) I did not take the purse. (2) Margaret knows who did it. (3) My daddy is rich and I have a purse of my own.
    Lillian: (1) I didn't take the purse. (2) I have never stolen anything. (3) Theo did it.
    Margaret: (1) I didn't take the teacher's purse. (2) Judy is guilty. (3) David can vouch for me because we've been friends our whole life.

    Later, each child admitted that two of their statements were true and one was false. Assuming this is true, who stole the purse?

    a. David b. Theo c. Judy d. Lillian e. Margaret

**SHORT ANSWER**
Place the answer in the appropriate space.

66. What is the least value of *y* that satisfies the inequality $|4+x|+|5+y|\le 80$?

67. A collection of 62 coins consists of $D$ dimes, $N$ nickels, and $Q$ quarters. The total value is \$8.30. What is the sum of all possible values of $N$?

68. A Pythagorean triple is a set of three positive integers that form the sides of a right triangle. For example, 3-4-5 is a Pythagorean triple. What is the largest positive integer that is **NOT** the shortest leg of a Pythagorean triple?

69. What is $|a-b|$ if $a$ and $b$ are the two real solutions of $f\left(f\left(x\right)\right)=1$ for $f\left(x\right)=2x^2+28x+91$?

70. Let $\frac{p}{q}$ represent the probability of rolling a sum of ten with three, fair six-sided dice. Assume $\frac{p}{q}$ is in reduced fractional form. What is $p+q$?

1. B
2. C
3. A
4. D
5. B
6. B
7. A
8. D
9. E
10. C
11. E
12. D
13. A
14. B
15. C
16. E, Correct answer is 1365.
17. E
18. A
19. D
20. C

66. − 85
67. 234
68. 4
69. 2
70. 9