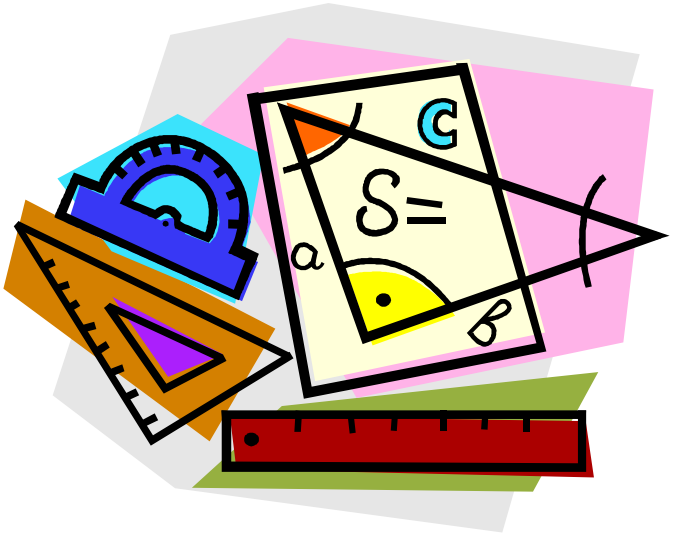
Comprehensive



Do NOT open until you are told to do so.

March 23, 2023

1. A meteorologist predicts a 40% chance of rain each day for the next five days. If the meteorologist is perfectly accurate, what is the probability to the nearest percent that there will be rain on at least one day of the five days?

   a. 92% b. 94% c. 60% d. 88% e. 50%

2. If ΔABC is inscribed in a semicircle whose diameter is AB, then which of the following is true?

   a. $AC + BC = AB$
   b. $AC + BC = AB\sqrt{2}$
   c. $AC + BC \geq AB\sqrt{2}$
   d. $AC + BC \leq AB\sqrt{2}$
   e. $AC + BC = (AB)^2$

3. If the graphs of the functions $f(x) = b(x-m)^2 + n$ and $g(x) = x - m$ intersect, then what is the greatest possible value of the product of b and n?

   a. $\frac{1}{4}$ b. 1 c. $\frac{3}{4}$ d. $\frac{1}{2}$ e. 2

4. What is the sum of all zeroes (both real and complex) of the function $f(x) = \frac{x^3 - 0.125}{x - 0.5}$?

   a. $-0.5 + 0.5i\sqrt{3}$ b. -0.5 c. 0.5 d. $-0.5 - 0.5i\sqrt{3}$ e. 0

5. If the discriminant of $ax^2 + 2bx + c = 0$ is zero for real numbers a, b, and c, then which of the following is always a true statement about a, b, and c?

   a. They are all negative numbers.
   b. They are all positive numbers.
   c. Only b is negative, and a and c are positive.
   d. They are consecutive terms of an arithmetic sequence.
   e. They are consecutive terms of a geometric sequence.

6. Two congruent parallel chords are 8 inches apart in a circle of radius 8 inches. What is the area in square inches of the region enclosed by the two chords and the circle?

   a. $\frac{64\pi}{3} + 32\sqrt{3}$ b. $32\sqrt{3} + \frac{128\pi}{3}$ c. $16\sqrt{3} + \frac{64\pi}{3}$ d. $16\sqrt{3} + \frac{128\pi}{3}$ e. $\frac{64\pi}{3} - 32\sqrt{3}$

7. What is $\sec\left(\arcsin\left(-\frac{15}{17}\right)\right)$?

   a. $\frac{17}{12}$ b. $\frac{17}{8}$ c. $-\frac{17}{8}$ d. $-\frac{17}{15}$ e. $\frac{17}{15}$

8. The relationship between the graphs of $y = \sin(x)$ and $y = \cos(x)$ is best described by which of the following?

   a. They are inverses of each other.
   b. They are reflections of each other.
   c. One is a dilation of the other.
   d. They differ by a horizontal shift of π.
   e. None of these.

9. Let $f(x,y) = \frac{x}{|x|} + \frac{y}{|y|} + \frac{xy}{|xy|}$. What is the range of *f*?

   a. $(-\infty, \infty)$ b. $[1, \infty)$ c. {-3,-1,1,3} d. {-1,1,3} e. {-1,3}

10. Kara looks at a wall clock (with constant velocity hands) at 2:15. What is the angle between the hour-hand and the minute-hand?

a. 30° b. 5° c. 22.5° d. 17.5° e. 28°

11. Let A, B, and C be unique collinear points such that $AB = BC = 2$. Let P be a point that lies on the circle centered at B with radius 2 and , also, lies on the circle centered at C with radius 2. What is the measure of angle PAC in degrees?

a. $60°$ b. $120°$ c. $90°$ d. $30°$ e. $45°$

12. If $k = \log_a b$, then which of the following is equal to $\log_a\left(\sqrt[3]{b^2}\right) - \log_b\left(a^{-3}\right)$?

a. $\frac{2k^2-9}{3k}$ b. $\frac{2k}{3}$ c. $\frac{2k^2+9}{3k}$ d. $\frac{k^2-9}{3k}$ e. $\frac{k^2+9}{3k}$

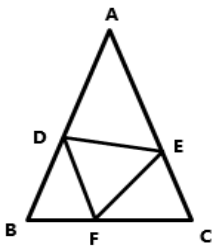
13. An arc length of 36 feet subtends a central angle of θ in a circle of diameter 15 feet. What is the measure of θ?

a. 4.8 rad b. 1.2 rad c. 2.4 rad d. 2.4π rad e. 4.8π rad

14. What is the maximum value of the function $P = 6x + 5y$ subject to the constraints: $x \geq 0$; $y \geq 0$; $x + y \leq 76$; and $x - 3y \geq 0$?

a. 380 b. 437 c. 456 d. 519 e. 446

15. Triangle ABC is isosceles with $AB = AC$. Equilateral triangle DEF is inscribed in triangle ABC, such that D is on side AB, E is on side AC, and F is on side BC. Let $m\angle BFD = a$, $m\angle ADE = b$, and $m\angle CEF = c$. Which of the following equations is true?



a. $2b = a + c$ b. $2b = a - c$ c. $2a = b - c$ d. $2a = b + c$ e. none of these

16. There are several pairs of integers (a,b) satisfying $a^2 - 4a + b^2 - 8b = 30$. What is the sum of the coordinates of all such points?

a. 60 b. 74 c. 72 d. 50 e. 0

17. A right triangle with hypotenuse of length 12 has area 4. What is its perimeter?

a. $12 + 2\sqrt{10}$ b. $12 + 2\sqrt{15}$ c. $12 + 2\sqrt{5}$ d. $12 + 4\sqrt{5}$ e. $12 + 4\sqrt{10}$

18. What is the sum of all real solutions to $\left(x^2 - 10x - 12\right)^{x^2+5x+2} = 1$?

a. -5 b. 25 c. 10 d. 15 e. 5

19. Let M be the sum of the solutions to $e^{-x}\sin(x) - e^{-x}\cos(x) = 0$, where $0 \le x \le 2\pi$. What is $\csc(M)$?

a. -1 b. $-\frac{2}{\sqrt{3}}$ c. 1 d. $\frac{2}{\sqrt{3}}$ e. undefined

20. An elementary school teacher had her purse stolen. The thief had to be Lillian, Judy, David, Theo, or Margaret. When questioned, each child made three statements.

    Lillian: (1) I didn't take the purse. (2) I have never stolen anything. (3) Theo did it.
    Judy: (1) I did not take the purse. (2) Margaret knows who did it. (3) My daddy is rich and I have a purse of my own.
    David: (1) I did not take the purse. (2) I did not know Margaret before I enrolled in this school. (3) Theo did it.
    Theo: (1) I am not guilty. (2) Margaret did it. (3) Lillian is lying when she says I stole the purse.
    Margaret: (1) I didn't take the teacher's purse. (2) Judy is guilty. (3) David can vouch for me because we've been friends our whole life.

    Later, each child admitted that two of their statements were true and one was false. Assuming this is true, who stole the purse?

    a. Lillian b. Judy c. David d. Theo e. Margaret

SHORT ANSWER

Place the answer in the appropriate space on the back of the scantron.

66. Let set A = {0,1,2,3,4,5,6,7,8,9}. How many three element subsets of set A contain at least two consecutive integers?

67. A Pythagorean triple is a set of three positive integers that form the sides of a right triangle. For example, 3-4-5 is a Pythagorean triple. What is the largest positive integer that is **NOT** the shortest leg of a Pythagorean triple?

68. What is $|a-b|$ if a and b are the two real solutions of $f\left(f\left(f\left(x\right)\right)\right)=1$ for $f\left(x\right)=2x^2+28x+91$?

69. How many ordered lists $\left(a,b,c,d,e,f\right)$ of nonnegative integers satisfy $a+b+c+d+e+f=8$?

70. In parallelogram $ABCD$, $\overline{BC}$ is extended beyond point C to point E. Points F and G are the points of intersection of $\overline{AE}$ with $\overline{BD}$ and $\overline{CD}$, respectively. If FG = 12 and EG =15, what is AF?

1. A
2. D
3. A
4. B
5. E
6. A
7. B
8. E
9. E
10. C
11. D
12. C
13. A
14. C
15. D
16. C
17. E
18. D
19. A
20. B

66. 64
67. 4
68. $\sqrt{2}$
69. 1287
70. 18