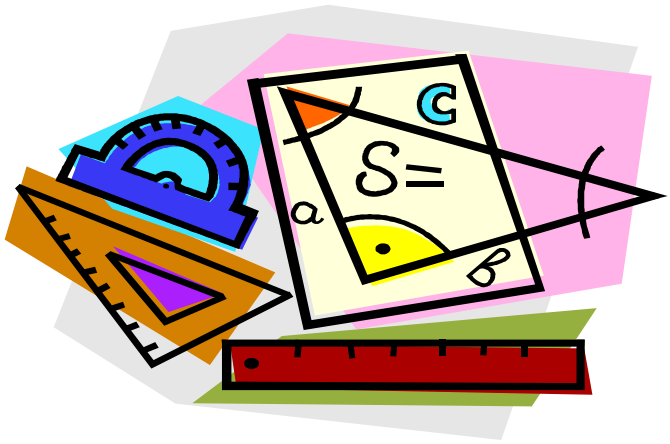
Level II



Do NOT open until you are told to do so.

March 23, 2023

1. A meteorologist predicts a 40% chance of rain each day for the next three days. If the meteorologist is perfectly accurate, what is the probability to the nearest percent that there will be rain on at least one day of the three days?

   a. 85% b. 87% c. 60% d. 78% e. 58%

2. If the hypotenuse of an isosceles right triangle is *x*, then what is the area of the triangle?

   a. $\frac{x}{2}$ b. $\frac{x}{4}$ c. $\frac{x^2}{4}$ d. $\frac{x^2}{2}$ e. x^2

3. Which statement negates the statement "All men are good golfers."?

   a. All women are good golfers.
   b. Some women are good golfers.
   c. No men are good golfers.
   d. All men are bad golfers.
   e. At least one man is not a good golfer.

4. Kara looks at a wall clock (with constant velocity hands) at 2:15. What is the angle between the hour-hand and the minute-hand?

   a. 22.5° b. 5° c. 30° d. 17.5° e. 28°

5. The average score on the midterm was 80 in a certain class. One student scored 52 on the midterm and decided to drop the class. This increased the average score to 84 for the remaining students. How many students remain in the course?

   a. 10 b. 8 c. 7 d. 5 e. 12

6. Ella observes it takes 20 seconds for a ferris wheel to complete one revolution. When she rides it, she is 20 ft away from the central axel of the wheel. How many feet will she travel while riding the wheel for one minute?

   a. 120π b. 60π c. 120 d. 60 e. 40π

7. What is the area of the triangle with vertices $(1,0)$, $(2,2)$, and $(4,3)$?

   a. 2 b. $\frac{5}{2}$ c. 1 d. $\frac{7}{2}$ e. $\frac{3}{2}$

8. A manufacturer built a machine that addresses 500 envelopes in 8 minutes. He plans to build another machine so that together the two machines will be able to address 500 envelopes in two minutes. How fast does the second machine need to address 500 envelopes?

   a. 3 min b. $\frac{5}{3}$ min c. $\frac{8}{3}$ min d. $\frac{7}{3}$ min e. $\frac{10}{3}$ min

9. What is the surface area of a cube inscribed in a sphere of radius 10 cm?

   a. 600 cm^2 b. 800 cm^2 c. 1000 cm^2 d. $600\sqrt{3}$ cm^2 e. $800\sqrt{3}$ cm^2

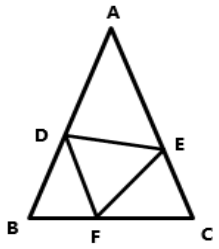
10. In rectangle $ABCD$, point E is on side $\overline{AB}$ so that AE = 10 and EB = 5. What fraction of the area of the rectangle is inside triangle AEC?

    a. $\frac{1}{3}$ b. $\frac{1}{2}$ c. $\frac{1}{4}$ d. $\frac{2}{3}$ e. $\frac{1}{6}$

11. How many rectangular solids are possible with a volume of 200 cubic meters and dimensions of integral value?

a. 5 b. 12 c. 16 d. 18 e. 36

12. Triangle ABC is isosceles with $AB = AC$. Equilateral triangle DEF is inscribed in triangle ABC, such that D is on side AB, E is on side AC, and F is on side BC. Let $m\angle BFD = a$, $m\angle ADE = b$, and $m\angle CEF = c$. Which of the following equations is true?



a. $2b = a + c$ b. $2b = a - c$ c. $2a = b - c$ d. $2a = b + c$ e. none of these

13. Let A, B, and C be unique collinear points such that $AB = BC = 2$. Let P be a point that lies on the circle centered at B with radius 2 and , also, lies on the circle centered at C with radius 2. What is the measure of angle PAC in degrees?

a. $60°$ b. $120°$ c. $90°$ d. $45°$ e. $30°$

14. Let a and b represent the legs of a right triangle, and let c represent the hypotenuse. If $a + b = \sqrt{65}$ and $c = \sqrt{33}$, what is the area of the triangle?

a. $4\sqrt{3}$ b. 10 c. 8 d. 32 e. 16

15. If ΔABC is inscribed in a semicircle whose diameter is AB, then which of the following is true?

a. $AC + BC = AB$

b. $AC + BC = AB\sqrt{2}$

c. $AC + BC \geq AB\sqrt{2}$

d. $AC + BC \leq AB\sqrt{2}$

e. $AC + BC = (AB)^2$

16. Find all the solutions to the equation $2x^2 - 12x - 18 = 0$.

a. $9, -3$ b. $-9, 3$ c. $3 \pm 3\sqrt{2}$ d. $3 \pm 2\sqrt{3}$ e. $6 \pm 6\sqrt{2}$

17. A right triangle ABC has an area of 4, and the hypotenuse AC is 12. What is the perimeter of ABC?

a. $12 + 4\sqrt{10}$ b. $12 + 2\sqrt{15}$ c. $12 + 2\sqrt{5}$ d. $12 + 4\sqrt{5}$ e. $12 + 2\sqrt{10}$

18. In a movie theater line, x people are behind Mark, who is y places in front of Sam. If there are z people in front of Sam, how many people are in line?

a. $x - y + z + 1$ b. $x - y + z + 2$ c. $x + y - z$ d. $x - 2y + z + 1$ e. $x - y + z$

19. In parallelogram $ABCD$, $\overline{BC}$ is extended beyond point C to point E. Points F and G are the points of intersection of $\overline{AE}$ with $\overline{BD}$ and $\overline{CD}$, respectively. If FG = 12 and EG =15, what is AF?

a. 16 b. 18 c. 20 d. 24 e. 27

20. An elementary school teacher had her purse stolen. The thief had to be Theo, David, Lillian, Judy, or Margaret. When questioned, each child made three statements.

Theo: (1) I am not guilty. (2) Margaret did it. (3) Lillian is lying when she says I stole the purse.
David: (1) I did not take the purse. (2) I did not know Margaret before I enrolled in this school. (3) Theo did it.
Lillian: (1) I didn't take the purse. (2) I have never stolen anything. (3) Theo did it.
Judy: (1) I did not take the purse. (2) Margaret knows who did it. (3) My daddy is rich and I have a purse of my own.
Margaret: (1) I didn't take the teacher's purse. (2) Judy is guilty. (3) David can vouch for me because we've been friends our whole life.

Later, each child admitted that two of their statements were true and one was false. Assuming this is true, who stole the purse?

a. Theo b. David c. Lillian d. Judy e. Margaret

SHORT ANSWER
Place the answer in the appropriate space.

66. What is the largest number of pieces into which a circular pizza can be cut with 8 straight cuts?

67. How many interior diagonals can be drawn in a regular 13-sided polygon?

68. The point $A(12,-5)$ is reflected across the x-axis to the point A'. The point A' is reflected across the y-axis to the point A''. What is the length of the line segment connecting point A and A''?

69. The widths and lengths of two distinct rectangles in yards form a sequence of four consecutive odd integers. The perimeter of the first rectangle is 44 yards less than twice the perimeter of the second rectangle, and the sum of their areas is less than 150 square yards. What is the sum of their areas?

70. A Pythagorean triple is a set of three positive integers that form the sides of a right triangle. For example, 3-4-5 is a Pythagorean triple. What is the largest positive integer that is **NOT** the shortest leg of a Pythagorean triple?

1. D
2. C
3. E
4. A
5. C
6. A
7. E
8. C
9. B
10. A
11. B
12. D
13. E
14. C
15. D
16. C
17. A
18. A
19. B
20. D

66. 37
67. 65
68. 26
69. 122
70. 4