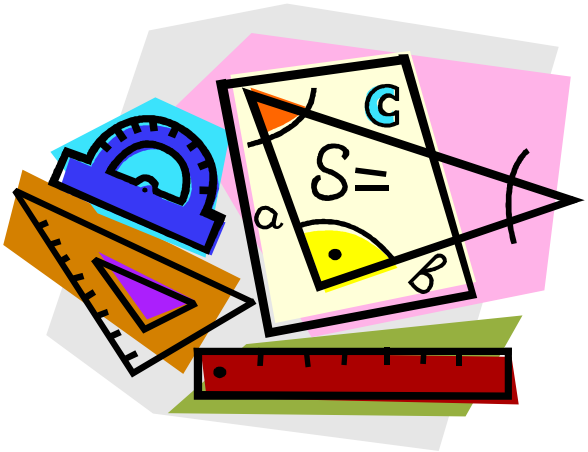
Level I



Do NOT open until you are told to do so.

March 23, 2023

1. A meteorologist predicts a 40% chance of rain each day for the next two days. If the meteorologist is perfectly accurate, what is the probability to the nearest percent that there will be rain on at least one day of the two days?

   a. 80% b. 57% c. 64% d. 75% e. 60%

2. In a movie theater line, x people are behind Mark, who is y places in front of Sam. If there are z people in front of Sam, how many people are in line?

   a. $x-y+z+1$ b. $x-y+z+2$ c. $x+y-z$ d. $x-2y+z+1$ e. $x-y+z$

3. If $a=\dfrac{2b+1}{b-1}$ for all real numbers such that $b\neq 1$, then which of the following is an expression for b ?

   a. $\dfrac{a-1}{2a+1}$ such that $a \neq -\frac{1}{2}$

   b. b. $\dfrac{2a+1}{a-2}$ such that $a \neq 2$

   c. c. $\dfrac{a-1}{a+2}$ such that $a \neq -2$

   d. d. $\dfrac{2a-1}{a+1}$ such that $a \neq -1$

   e. e. $\dfrac{a+1}{a-2}$ such that $a \neq 2$

4. Add consecutive positive integers beginning with one until the sum exceeds 1000. What was the last integer that was added?

   a. 43 b. 50 c. 42 d. 45 e. 51

5. If a megabyte is 2^{20} bytes, a gigabyte is 2^{30} bytes, and a music file can store one minute of music per megabyte, then how many days of music can be stored on a 45-gigabyte portable media player?

   a. 42 b. 32 c. 16 d. 64 e. 40

6. Let $f(x)=\dfrac{3x+1}{2x+3}$ for all real numbers such that $x \neq -\frac{3}{2}$. What is $f\left(\dfrac{1}{x}\right)$?

   a. $\dfrac{3x+1}{2x+3}$ b. $\dfrac{3x+2}{x+3}$ c. $\dfrac{x+2}{2x+3}$ d. $\dfrac{x+3}{3x+2}$ e. $\dfrac{2x+3}{3x+1}$

7. What is the value of x in the solution to the system: $\begin{matrix}2x+3y=1\\4x-5y=24\end{matrix}$?

   a. 3 b. 2.5 c. 4 d. 5 e. 3.5

8. The average score on the midterm was 80 in a certain class. One student scored 59 on the midterm and decided to drop the class. This increased the average score to 83 for the remaining students. How many students remain in the course?

   a. 10 b. 8 c. 7 d. 5 e. 12

9. A line with slope 6 intersects the curve $y=x^2$ at two distinct points. If the y-coordinate of one point is 4 times the y-coordinate of the other point and the two points do not lie in the same quadrant, then what is the sum of the y-coordinates?

   a. 20 b. 180 c. 125 d. 80 e. 245

10. Ken sold a house and a store for $120,000 each. The house sold at a gain of 20% of its value, and the store sold at a loss of 20% of its value. What was the net change on the entire transaction?

    No loss or gain

    b. A loss of $10,000

    c. A gain of $10,000

    d. A gain of $30,000

    e. A loss of $30,000

11. How many ounces of water are required to reduce 9 ounces of shaving lotion containing 50% alcohol to a lotion containing 30% alcohol?

    a. 4.5 b. 5 c. 8 d. 6 e. 3

12. A certain redwood tree grew to 60% of its current height in its first century of life. In its second century, it increased its first century height by one third. Since then it has added another 50 feet to its stature. How many feet tall is this redwood today?

    a. 250 ft b. 300 ft c. 150 ft d. 200 ft e. 325 ft

13. A manufacturer built a machine that addresses 500 envelopes in 8 minutes. He plans to build another machine so that together the two machines will be able to address 500 envelopes in two minutes. To reach this goal, how long would it take the second machine (on its own) to address 500 envelopes?

    a. 3 min b. $\frac{5}{3}$ min c. $\frac{8}{3}$ min d. $\frac{7}{3}$ min e. $\frac{10}{3}$ min

14. How many irrational solutions does the equation $x^4 + 6x^3 - 8x - 48 = 0$ have?

    a. 0 b. 1 c. 2 d. 3 e. 4

15. A bicyclist riding against the wind averages 12 miles per hour traveling from home to work, but with the wind averages 18 miles per hour returning from work to home. Assuming a constant wind speed, what is his average speed for the round trip?

a. 14 mph b. 14.4 mph c. 14.5 mph d. 14.6 mph e. 15 mph

16. Let $f(x)=|5x+3|-|3x-5|$. What is the minimum value of $f(x)$?

a. $\frac{34}{5}$ b. $\frac{40}{3}$ c. 0 d. $-\frac{34}{5}$ e. $-\frac{40}{3}$

17. Which statement negates the statement "All men are good golfers."?

a. All women are good golfers.

b. b. Some women are good golfers.

c. c. No men are good golfers.

d. d. All men are bad golfers.

e. e. At least one man is not a good golfer.

18. Let $f(x,y)=\frac{x}{|x|}+\frac{y}{|y|}+\frac{xy}{|xy|}$. What is the range of *f*?

a. $(-\infty,\infty)$ b. $[1,\infty)$ c. {-3,-1,1,3} d. {-1,1,3} e. {-1,3}

19. Let a and b represent the legs of a right triangle and let c represent the hypotenuse. If $a+b=\sqrt{65}$ and $c=\sqrt{33}$, what is the area of the triangle?

a. $4\sqrt{3}$ b. 10 c. 8 d. 32 e. 16

20. An elementary school teacher had her purse stolen. The thief had to be Judy, David, Lillian, Theo, or Margaret. When questioned, each child made three statements.

    Judy: (1) I did not take the purse. (2) Margaret knows who did it. (3) My daddy is rich and I have a purse of my own.
    David: (1) I did not take the purse. (2) I did not know Margaret before I enrolled in this school. (3) Theo did it.
    Lillian: (1) I didn't take the purse. (2) I have never stolen anything. (3) Theo did it.
    Theo: (1) I am not guilty. (2) Margaret did it. (3) Lillian is lying when she says I stole the purse.
    Margaret: (1) I didn't take the teacher's purse. (2) Judy is guilty. (3) David can vouch for me because we've been friends our whole life.

    Later, each child admitted that two of their statements were true and one was false. Assuming this is true, who stole the purse?

    a. Judy b. David c. Lillian d. Theo e. Margaret

SHORT ANSWER
Place the answer in the appropriate space.

66. The number of seconds in 6 weeks equals $n!$. What is n?

67. What is the largest integer that evenly divides each of 252, 378, 945?

68. A barge loads trailers from semi-trucks for overseas shipments. The barge can carry trailers stacked 10 deep, 9 wide, and 3 tall. A crane loads one trailer at a time and it takes an average of 44 seconds per trailer to load the barge to capacity with trailers. If the barge begins loading trailers at 3:32 pm, what time will it finish loading?

69. Start with a temperature measured in degrees Fahrenheit and then add 50°F to it. Convert both starting and ending temperatures to degrees Celsius. What Fahrenheit temperature must you start with so that the ending Celsius temperature is twice the starting Celsius temperature? (hint: 0°C = 32°F and 100°C = 212°F)

70. As a bonus for signing up with U-Tunes, you get to choose any three of this week's top twenty songs to download for free. In how many ways can you make this selection?

1. C
2. A
3. E
4. D
5. B
6. D
7. E
8. C
9. B
10. B
11. D
12. A
13. C
14. A
15. B
16. D
17. E
18. E
19. C
20. A

66. 10
67. 63
68. 6:50 PM
69. 82°F
70. 1140