Connecting Industry to Mathematics Instruction

NSF ATE Award # 1954291

In partnership with

Crests and Sags
Student Activity Sheet

Quantity: Flooding and Sight Distance

The North Carolina Department of Transportation (NCDOT) Location and Surveys Unit provides many customer services including the geometric design of our highway system. There have been several reports of flooding on a particular portion of highway causing hydroplaning of vehicles. Additionally, on a different highway, there have been a record number of car accidents where drivers claim they could not see an object in the road. You have been tasked with the following: 1. To determine if the roadway drainage is placed appropriately on the first highway and what changes need to be made to decrease the number of hydroplaning vehicles, and 2. To determine if any changes need to be made to the second highway that would decrease the number of accidents claimed.

Roads are designed with vertical curves to gently transition between the changes in elevation of the terrain. Vertical curves are the curves that go up and down through hills and valleys. "Sag" vertical curves are an area of concern for hydroplaning accidents due to standing water. Typically, NCDOT puts a drainage device at the low point of the sag. "Crest" vertical curves have their own problems in that drivers need to be able to stop in an appropriate amount of time if an object obstructs the road. Properly designed vertical curves consider the driver's eye height and a desirable stopping sight distance to an obstacle in the road.

Important Terminology

- Station: 100 feet.
  - Projects start at station 0+00
  - Stations are written as the number of hundreds + remaining. Ex: The first station is 1+00 and the second station would be written as 2+00.
  - If you are 11+ 62.5 stations from the start of a project, you are 1162.5 feet from the start of the project.
- Slope: $\frac{\text{rise}}{\text{run}}$ (use feet for horizontal distance)

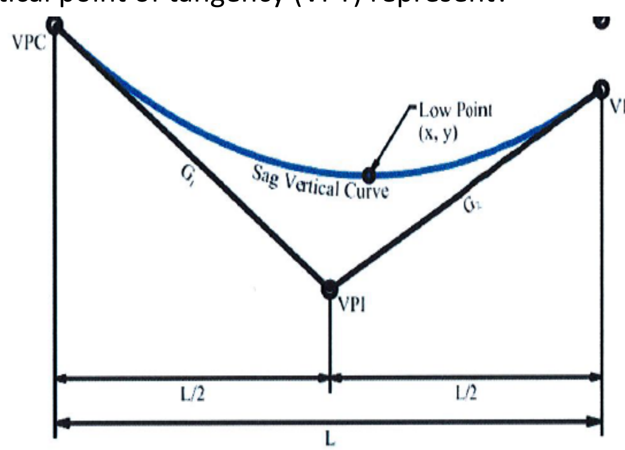
- Grade: $\frac{\text{rise}}{\text{run}} \times 100\%$ (use stations for horizontal distance)
  - 3% grade means for every 100 feet horizontal, the rise in the road is 3 feet vertically
  - 3% = .03, but using stations would be reported at a grade of 3

Preliminary Discussion Questions for Task 1:

1. After watching the launch video and completing the Desmos Activity, identify which picture represents a sag curve and which picture represents a crest curve. Explain your reasoning.

2. Given the three pictures below, provided by the NCDOT, which one do you believe would have more flooding associated? Explain what factors could cause flooding.

3. What factors should be taken into consideration so that there is minimal flooding?
4. Using the picture below, answer the following questions.
   a. What would the values of G_1 and G_2 represent? How would they affect the shape of the curve?
   b. What do you notice about the low point on the curve and the vertical point of intersection (VPI)? Why do you believe they are not the same?
   c. If L represents the horizontal length of the vertical curve, what do the vertical point of curvature (VPC) and the vertical point of tangency (VPT) represent?



5. In looking at the given figure in question 3, what type of function would represent the curve?
6. If the beginning grade is -4.1% and the ending grade is 2.6%, are you dealing with a "sag" curve or a "crest" curve? Explain your reasoning.
7. How would one determine the horizontal distance to the minimum or maximum elevation of the curve? Determine a generic formula to compute this value, labeled as x_m in terms of L, G_1, and G_2, where $a = \frac{G_2 - G_1}{2L}$ in $Y = Y_{VPC} + G_1 x + ax^2$.

Task 1 Questions:

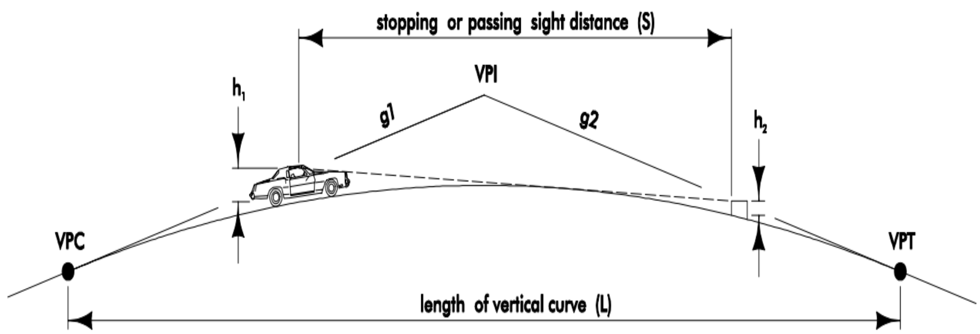
8. You have a 400-foot vertical curve (horizontal width) with a VPI station at 14+24.08 and an elevation of 104.77 feet (vertical distance) provided the beginning grade is -5.1% and ending grade is 2.4%.
   a. Draw a picture of the scenario above listing all important information provided on your picture.
   b. Determine the vertical elevation for the VPC (y-intercept) using the equation of a line.
   c. Derive a curve elevation function, $Y = Y_{VPC} + G_1 x + ax^2$. Use x in ***horizontal feet*** from the VPC and y as the elevation above sea level.
   d. Either use what you know about the relationship between grades and stations to rewrite the function or derive a curve elevation function, $Y = Y_{VPC} + G_1 x + ax^2$ where x ***in horizontal stations*** from the VPC and y as the elevation above sea level.
9. Calculate elevations for stations using the station interval of 100 feet and starting with the first even station. (Hint: Even station would be 14 + 00 and odd station would be 15 +00)

10. Suppose in this scenario the stormwater drop is placed 1.4 stations from the VPC. Without making any computations, do you think this is the optimal placement for the roadway drainage? Explain.
11. If you answered question 5 with a no, where should the stormwater drop be placed for optimal roadway drainage? Determine the elevation for the appropriate storm drain placement.
12. During a hurricane event, the model predicts stormwater will rise to an elevation of 109.00 feet. Determine the interval, in stations, in which the roadway floods.
13. Why do you think the roadway will still flood even though the drain is at its optimal location for roadway drainage? What other factors may be contributing to the flooding issues?

Preliminary Discussion Questions for Task 2:

Note: $G_1 = g_1$ and $G_2 = g_2$.

14. Given the first figure below, what do the values of h_1 and h_2 represent? Note that these distances must be in feet. (Hint: Think about how h_1 relates to the car in front. The box in front of h_2 represents an object in the road.)



15. Solve $L = \dfrac{AS^2}{\left(\sqrt{2h_1} + \sqrt{2h_2}\right)^2}$ given $A = | G_2 - G_1 |$ is the absolute difference in tangent grades, when $S < L$ for S. How many formulas result? Can we eliminate one? Why or why not?

16. Without making any computations, what happens to the sight distance, S , as the height of the object in the road gets larger? Explain your reasoning.
17. Explain how the driver's height would affect the sight distance. Why do you believe the American Association of State Highway and Transportation Officials (AASHTO) sets their standard for the driver's eye height to 3.5 feet?

Task 2 Questions

18. The average height of a deer is between 21 inches and 42 inches tall.
    a. Without making any calculations, which deer height would give you the greatest sight distance? Explain your reasoning.
    b. Using AASHTO's recommended value for driver's eye height, determine the sight distance for a 750-foot vertical curve with a start grade of 2.0% and an end grade of -3.0% for the average shortest and tallest deer assuming $S < L$. (Hint: Use the equation you found in number 15 and

remember that when you are using grades your horizontal distance must be in stations and vertical distances are measured in feet.)

19. Is the assumption $S < L$ met for each case above? Write the inequality.

20. Given the stopping sight distance in feet is given by $S = 1.47V(2.5) + \dfrac{V^2}{30\left[0.347826 + \left(\dfrac{|G|}{100}\right)\right]}$ where V the design speed in miles per hour, G is the grade (%), and the speed is currently set to 70 miles per hour on this stretch of road, answer the following:
    a. Compute the stopping sight distance with a grade going into the curve upwards of 2.0%.
    b. Compute the stopping sight distance with a grade going out of the curve downwards of 3.0%.
    c. How do the values in parts a and b relate to the sight distances found in question 18?
    d. Is the design speed appropriate for this stretch of road? Explain why or why not.
21. Would you recommend a new speed limit in this situation? If so, determine the new design speed for this situation using the following formula in problem 20.
22. Look at the graph of Length of Vertical Curve versus Algebraic difference in grades below. Does your recommended speed limit match what this graph suggests? If not, what error could have possibly happened causing a mismatch?

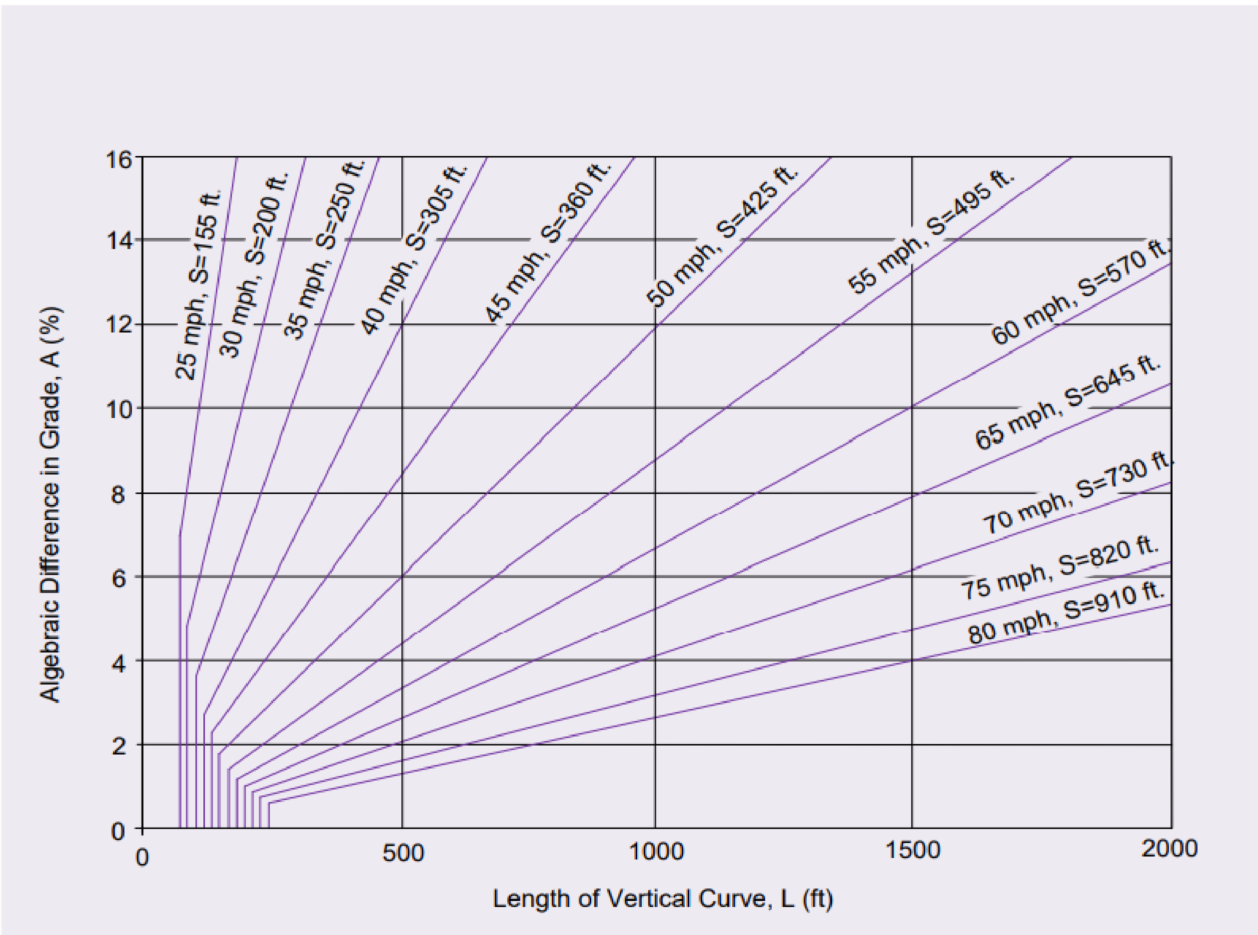


Figure taken from https://wsdot.wa.gov/publications/manuals/fulltext/M22-01/1260.pdf