# Level Three
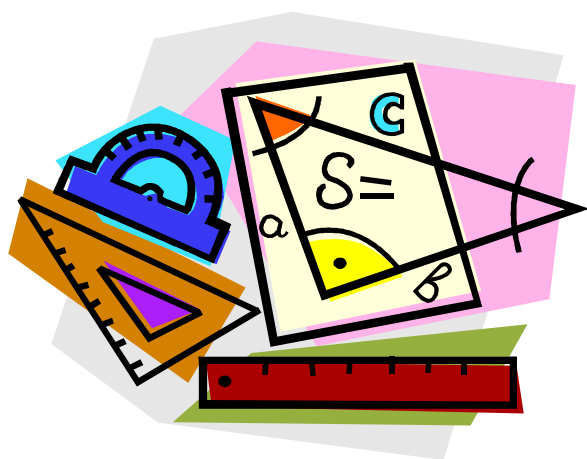
Do NOT open until
you are told to do so.

1.  A drawer contains 16 socks, 6 of which are orange.  When three socks are drawn (without replacement) at random, what is the probability that at least one sock is orange?

    a.  $\frac{3}{8}$
    b.  $\frac{5}{8}$
    c.  $\frac{3}{14}$
    d.  $\frac{11}{14}$
    e.  $\frac{17}{56}$

2.  Suppose that for all $x \neq 1, -5, f\left(\frac{2x-3}{x+5}\right) = \frac{3x+2}{x-1}$.  Determine $f(x^2)$.

    a.  $\frac{9x^2+12x+4}{x^2-2x+1}$
    b.  $\frac{13x^2+13}{6x^2+1}$
    c.  $\frac{4x^2-12x+9}{x^2+10x+25}$
    d.  $\frac{3x^2+2}{x^2-1}$
    e.  $\frac{6x^2-5x-6}{x^2+4x-5}$

3.  In triangle $ABC$, $AB = 2, BC = 1$.  Suppose side $AC$ and the median from point $B$ to side $AC$ have the same length.  What is the value of $(AC)^2$ ?

    a.  $\frac{3}{2}$
    b.  2
    c.  $\frac{9}{4}$
    d.  3
    e. none of these

4.  Which of the following is a simplification of $(\log_4 9)(\log_3 7)(\log_7 8)$ ?

    a.  3
    b.  6
    c.  12
    d.  $\log_{14} 24$
    e.  $\log_{84} 504$

5.  Right triangle $ABC$ is such that angle $B$ is a right angle and $BC = 60$.  Let point $D$ be on segment $AB$ such that $AD = 1$.  Segment $BD$ is the diameter of a semicircle that is tangent to segment $AC$ at point $E$.  Determine the area of the semicircle.

    a.  $30\pi$
    b.  $36\pi$
    c.  $60\pi$
    d.  $72\pi$
    e.  $144\pi$

6.  Aziza runs a dragon fruit delivery service.  She charges $4 for each dragon fruit and a $1 delivery fee.  Aziza creates data set A from how many dragon fruit were ordered in each delivery, and data set B from how much she charged for each delivery.  What is the ratio of the standard deviation of data set B to the standard deviation of data set A?

    a.  1
    b.  2
    c.  4
    d.  5
    e.  none of these

7.  Let $x$ be the tens digit and let $y$ be the ones digit of $2024^{2024}$.  Determine the remainder when $10x + y$ is divided by 5.

    a.  0
    b.  1
    c.  2
    d.  3
    e.  4

8.  Suppose a triangle in the $xy$-plane has vertices at $(0,0), (6,0)$, and $(0,8)$.  Let $S$ be the set of points within 1 unit of the triangle's vertices or edges.  What is the area of $S$?

    a.  $\pi + 30$
    b.  $\pi + 36$
    c.  $\pi + 42$
    d.  $\pi + 48$
    e.  $\pi + 50$

9.  Calculate the exact value of $\cos^{-1}\left(\sin\left(\frac{7\pi}{5}\right)\right)$

    a.  $-\frac{\pi}{10}$
    b.  $\frac{\pi}{10}$
    c.  $\frac{2\pi}{5}$
    d.  $\frac{3\pi}{5}$
    e.  $\frac{9\pi}{10}$

10. The populations of Treyville and Sethburg were equal in 1990. After 30 years of change, the populations were equal again in 2020. From 1990 to 2000, the population of Treyville increased by 80%. In the next decade, it increased by 60%. In the following decade, the population decreased by 40%. The population of Sethburg increased by the same percentage each decade. What was that percentage?

    a. 12.0%          b. 20.0%          c. 24.2%          d. 57.6%          e. 72.8%

11. Evaluate $\left(\frac{2}{1+\sqrt[3]{2}+\sqrt[3]{4}} + 2\right)^3$

    a. 8          b. $\frac{64}{7}$          c. 16          d. 32          e. 64

12. Suppose the expression $\sqrt[4]{a\sqrt[3]{b\sqrt{c}}}$ is re-written in the form $\sqrt[24]{a^x b^y c^z}$, where $a, b, c, x, y, z$ are positive integers. Determine the value of $x + y + z$.

    a. 7          b. 9          c. 12          d. 24          e. 26

13. Determine the constant term in the expansion of $(2x^3 + x^{-4})^7$ ?

    a. 84          b. 128          c. 448          d. 560          e. 672

14. At a high school, 25% of the students are in 9th grade, and half of the high school students eat the cafeteria food. If 20% of the students who each the cafeteria food are in 9th grade, then what proportion of students in the 9th grade eat the cafeteria food?

    a. 20%          b. 25%          c. 30%          d. 40%          e. 50%

15. Suppose the product of the solutions of $(x + 1)^2 + (x + 2)^2 + (x + 3)^2 = a$ equals 3, where $a$ is a positive integer. Determine the value of $a$.

    a. 3          b. 5          c. 6          d. 7          e. 9

16. Suppose $T(t) = 125e^{-kt} + 75°\text{F}$ is the temperature of a pot of coffee $t$ minutes after it is brewed. If the pot's lid is on, $k = 0.02$, and if the pot's lid is off $k = 0.1$. Let $t_{\text{on}}$ and $t_{\text{off}}$ be the different times required for the pot to cool to 150°F with the lid on and off, respectively. What is $\frac{t_{\text{on}}}{t_{\text{off}}}$ ?

    a. $\frac{1}{20}$          b. $\frac{1}{5}$          c. $\frac{5}{3}$          d. 2          e. 5

17. How many solutions does the equation $\sin(2x) = \cos(x)$ have on the interval $0 < x < \pi$ ?

    a. 1          b. 2          c. 3          d. 4          e. 5

18.  Suppose a three-digit number is chosen at random and the number of distinct digits is counted (for example, the number 522 has 2 distinct digits).  What is the expected number for distinct digits?

    a.  2.439            b.  2.475            c.  2.5            d.  2.71            e.  2.75

19. Let $f(x) = \dfrac{4^x - 1}{4^x + 1}$  Determine the range of $f(f(x))$.

    a.  $(-0.6, 0.6)$        b.  $(-0.5, 0.5)$        c.  $(0, 0.5)$        d.  $(0, 0.6)$        e. none of these

20. In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false.  Four amphibians Alpha, Beta, Delta, and Epsilon live together in the swamp. They make the following statements:

    Alpha:  "There is at least one frog."
    Beta:    "Delta is a frog."
    Delta:   "If you ask Beta, Beta would say that Epsilon is a frog."
    Epsilon: "Alpha is a toad or Delta is a toad."

How many of the amphibians are frogs?

    a.  0            b.  1            c.  2            d.  3            e.  4

**SHORT ANSWER**
Place the answer in the appropriate space.

66. In the set of all positive integers less than 2024, how many numbers contain exactly two "0"s ?

67. If the length of the medians of triangle ABC are 5m, 12m, and 13m, then what is the area of triangle ABC in square meters?

68. A student toying with a calculator notices that 2424 is not divisible by 7, but 242,424 is divisible by 7. If the calculator has a 12-digit display, then starting with 1414 and ending with 949,494,949,494, how many numbers of the form a4a4 … a4 (where a is a digit from 1 to 9) will the student discover are divisible by 7?

69. What is the remainder when $1013^2$ is divided by $1011^2$ ?

70. Given the complex number $z = 1681 + ai$, where $a$ is a positive integer, what is the value of $a$, if $z^2$ and $z^3$ have equal imaginary parts?

2024 Wake Tech HS Level Three Test

1. D
2. B
3. B
4. A
5. D
6. C
7. B
8. C
9. E
10. B
11. C
12. B
13. D
14. D
15. B
16. E
17. C
18. D
19. A
20. B

66. 38
67. 40
68. 24
69. 4048
70. 2911