

1. A certain disease occurs in 1% of the male population and the test for it is 98% accurate (which means 98% of the time the test correctly identifies who does or who does not have the disease). If a man tests positive which of the following is closest to the chance that he truly has the disease?  
a. 98%                      b. 50%                      c. 30%                      d. 20%                      e. 10%
  
2. For how many rational values of  $x$  is  $P(x) = |x^2 - 28x + 160|$  a prime number?  
a. 0                      b. 1                      c. 2                      d. 3                      e. 4
  
3. A positive integer  $N$  leaves the same nonzero remainder  $r_1$  when divided into 527, 622, and 698. Another positive integer  $M$  leaves the same nonzero remainder  $r_2$  when divided into 736, 881, and 997. What is  $M + N$ ?  
a. 46                      b. 48                      c. 50                      d. 52                      e. 54
  
4. An infinite geometric series has a common ratio  $\frac{2}{3}$  and sums to the value  $S$ . If the first, third, and all other odd terms are doubled, and the second, fourth, and all other even terms are halved, what is the sum of the new series?  
a.  $40.6S$                       b.  $0.8S$                       c.  $1.4S$                       d.  $1.2S$                       e.  $1.0S$
  
5. If  $\sin(x)\cos(x) = 4\sin(x) + 4\cos(x)$ , then  $\sin(2x)$  can be represented in the form  $a + b\sqrt{c}$ , where  $a$ ,  $b$ , and  $c$  are integers and  $c$  is a prime number. What is  $a + b + c$ ?  
a. 41                      b. 45                      c. 49                      d. 53                      e. 57

6. How many of the following seven shapes have rotational symmetry with a positive angle measure less than  $360^\circ$ : rectangle, parallelogram, trapezoid, regular pentagon, triangle, circle, and hexagon?
- a. 2                      b. 3                      c. 4                      d. 5                      e. 6
7. A three digit number,  $ABC$ , is 29 times the sum of its digits. When the first digit is made the last digit the new number,  $BCA$ , is 68 times the sum of the digits. When the new first digit is made the last digit, the new number,  $CAB$ , is 14 times the sum of the digits. What is the sum of the squares of the digits?
- a. 38                      b. 37                      c. 43                      d. 41                      e. 39
8. In rectangle  $ABCD$ ,  $AB = 6$  and  $BC = 2$ . If  $E$  is between  $A$  and  $B$  such that  $AE = BC$  and  $F$  is the intersection of lines  $BD$  and  $CE$ , what is the area of  $\triangle BCF$ ?
- a. 2.2                      b. 2.4                      c. 2.6                      d. 2.8                      e. 3.0
9. Three faces of a rectangular box have a common point, which is a corner of the box. The centers of these faces are the vertices of a triangle with sides of length 4, 5, and 6 cm. What is the volume of the box?
- a.  $120\sqrt{2} \text{ cm}^3$       b.  $90\sqrt{6} \text{ cm}^3$       c.  $45\sqrt{6} \text{ cm}^3$       d.  $125 \text{ cm}^3$       e.  $45\sqrt{3} \text{ cm}^3$
10. What is the sum of all real numbers  $x$  such that  $(2^x - 4)^3 + (4^x - 2)^3 = (4^x + 2^x - 6)^3$ ?
- a. 2                      b.  $\frac{3}{2}$                       c. 3                      d.  $\frac{7}{2}$                       e.  $\frac{5}{2}$

11. A positive integer is a palindrome if the integer obtained by reversing the digits is equal to the original integer. How many years in the millennium between 1000 and 2000 are palindromes AND are the product of a 2-digit palindrome and a 3-digit palindrome?
- a. 6                      b. 5                      c. 9                      d. 10                      e. 11
12. Define  $f(x) = \sin(\pi x)$  and  $F(x) = (f(x))^2$ . What is  $f\left(f\left(\frac{1}{6}\right)\right) + F\left(F\left(\frac{1}{3}\right)\right)$ ?
- a. 1                      b.  $\frac{3}{4}$                       c.  $\frac{4}{3}$                       d.  $\frac{3}{2}$                       e. 2
13. If  $x + \sqrt{x^2 - 1} + \frac{1}{x + \sqrt{x^2 - 1}} = 20$ , then what is  $x^2 + \sqrt{x^4 + 1} + \frac{1}{x^2 - \sqrt{x^4 + 1}}$ ?
- a. 0                      b. 21                      c. 51.005                      d. 61.25                      e. 200
14. Different letters are placed on the 18 faces of 3 standard 6-sided dice, one per face. Choosing 1 letter from each die, the following words can be formed: bow, boy, cot, dry, gas, hat, oat, old, one, pay, pie, red, six. Which of the following could also be spelled?
- a. eat                      b. rap                      c. top                      d. wad                      e. won
15. If  $\{r, s, t, u, v\}$  satisfies the system  $\begin{cases} 3r + 10s + 16t + 30u + 25v = 10 \\ 4r + 15s + 20t + 36u + 36v = 11 \\ 5r + 20s + 24t + 42u + 49v = 20 \end{cases}$ , then what is the value of  $6r + 25s + 28t + 48u + 64v$ ?
- a. 33                      b. 34                      c. 37                      d. 36                      e. 35

16. If  $\log_3 10 = 2.095903$ , then how many digits does 100,000 have when expressed in base 3?
- a. 12                      b. 13                      c. 10                      d. 8                      e. 11
17. Let  $ax + b = 15$  and  $15x + a = b$  have the same unique solution, where  $a$  and  $b$  are positive integers both less than or equal to 30. What is the sum of all possible values of  $a$ ?
- a. 28                      b. 43                      c. 58                      d. 78                      e. 93
18. Three bags each contain two marbles. Bag A contains two blue marbles, bag B contains two red marbles, and bag C contains one red and one blue marble. Two bags are chosen at random and then one marble is taken from each bag. What is the probability that the marbles are the same color?
- a.  $\frac{3}{4}$                       b.  $\frac{1}{6}$                       c.  $\frac{1}{3}$                       d.  $\frac{2}{3}$                       e.  $\frac{5}{12}$
19. The vertices of a square in the first quadrant are  $(x, 0)$ ,  $(0, y)$ ,  $(a, b)$ , and  $(c, d)$ . What is  $x + y$ , if  $a + b = 19$  and  $c + d = 14$ ?
- a. 11                      b. 9                      c. 17                      d. 15                      e. 13
20. An isosceles triangle has equal sides of length  $A$  and the altitude to the third side is length  $B$ . Which of the following represents the radius of the circumscribed circle of the triangle?
- a.  $\frac{A^2}{2B}$                       b.  $\frac{B^2}{4A}$                       c.  $\frac{B\sqrt{2}}{2A}$                       d.  $\frac{A^2}{4B}$                       e.  $\frac{B^2}{2A}$

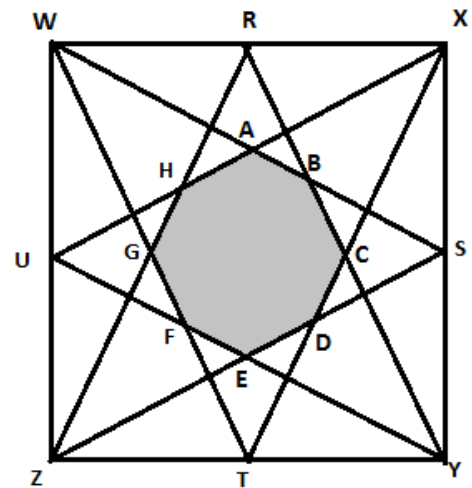
21. Which of the following expressions is identically equivalent to  $\sec(x+y)$ ?

- a.  $\frac{\cos(x+y)}{\cos^2 x - \sin^2 y}$     b.  $\frac{\cos(x-y)}{\cos^2 x - \sin^2 y}$     c.  $\frac{\cos(x-y)}{\cos^2 x + \sin^2 y}$     d.  $\frac{\cos(x+y)}{\cos^2 x + \sin^2 y}$     e.  $\frac{\cos(x-y)}{\sin^2 y - \cos^2 x}$

22. Mary and Jackie are “empty-nesters” (that means no children living at home). They decide that they should go out to dinner three nights a week for the next year. To keep things interesting they stipulate that they will not go to the same restaurant more than once in any week AND no one week may contain the same three restaurants as any other week. What is the minimum number of restaurants that make this possible?

- a. 10                                  b. 16                                  c. 12                                  d. 8                                  e. 20

23. In the diagram to the right, lines join the vertices of the square WXYZ to the midpoints of the sides (R, S, T, and U) to form the octagon ABCDEFGH (shaded). What is the ratio of the area of the square to the area of the octagon?



- a.  $4\sqrt{3}$                                   b.  $2\sqrt{3}$                                   c. 4                                  d.  $4\sqrt{2}$                                   e. 6

24. If you roll three fair six-sided dice, what is the probability that the sum of the three numbers rolled is prime?

a.  $\frac{71}{216}$

b.  $\frac{2}{3}$

c.  $\frac{1}{3}$

d.  $\frac{37}{108}$

e.  $\frac{73}{216}$

25. Five murder suspects, including the guilty party, are being interrogated by the police. Results of a polygraph indicate two of them are lying and three are telling the truth. If the polygraph results are correct, who committed the murder?

Suspect A: "Suspect D is the murderer."

Suspect B: "I am innocent."

Suspect C: "It wasn't Suspect E."

Suspect D: "Suspect A is lying."

Suspect E: "Suspect B is telling the truth."

a. E

b. C

c. B

d. A

e. D

**SHORT ANSWER**

Place the answer in the appropriate space.

66. If order doesn't matter, in how many ways can 2016 be written as the product of 3 positive integers?  
For example, one product would be  $1 \times 1 \times 2016 = 1 \times 2016 \times 1 = 2016 \times 1 \times 1$ .

67. A real valued function has the following properties:

A.  $f(0) \neq 0$

B.  $f(1) = 5$

C.  $f(x)f(y) = f(x+y) + f(x-y)$

What is  $f(5)$ ?

68. Pairs of integers in the range 1 to 10 inclusive are chosen randomly (assume the pair can be equal integers). What is the average positive difference between the two chosen integers?

69. From the series of the first one hundred odd integers,  $1+3+5+7+9+\dots+197+199$  it is possible to extract a sub-series which sums to almost any four digit power. For example:

$$3^7 = 2187 = 55 + 57 + 59 + \dots + 107$$

$$2^{12} = 4096 = 1 + 3 + 5 + \dots + 127$$

$$6^5 = 7776 = 37 + 39 + 41 + \dots + 179$$

What is the first term of such a sub-series which sums to  $5^5 = 3125$ ?

70. Bob and Pam are selling caps and T-shirts for the Math Club. Each item is priced in whole dollars and a T-shirt costs more than a cap. Bob remarked, "I just had two sales, one for \$39 and one for \$61." Pam said, "That can't possibly be right. There is no possible combination of caps and T-shirts that would sell for either amount." Bob checked and sure enough he was wrong in both cases. The discussion was interrupted when a customer who bought more than one item was correctly charged \$19. What is the cost of a T-shirt?

Answer Key

1. c
2. e
3. b
4. c
5. a
6. c
7. d
8. b
9. b
10. d
11. c
12. d
13. a
14. e
15. c
16. e
17. c
18. c
19. a
20. a
21. b
22. d
23. d
24. e
25. a

66. 66
67. 2525
68. 3
69. 101
70. 11