Connecting Industry to Mathematics Instruction
NSF ATE Award # 1954291

Stockpile - Student Activity Sheet

In partnership with
WAKE TECH
WAKE COUNTY PUBLIC SCHOOL SYSTEM
WakeEd PARTNERSHIP

How can you find the volume of a very large dirt pile on a construction project?

Part 1. Stockpile Boundary

Introduction: When a construction site is at the beginning phases, dirt is moved around for building construction projects on the site. Hauling large amounts of dirt off a site at the conclusion of a project is very expensive. To prevent this, dirt is moved around on the site from one place to another to flatten one area and fill in large holes of gaps in another area of the site. Surveyors use all types of technology to get volumes of dirt piles and to determine vacant space volumes to design plans for moving the dirt in an attempt to keep costs down. This activity introduces you to techniques that surveyors use to determine volumes of large piles of dirt.

1. Estimate the volume of the rock shown below within 10 seconds of first seeing it.
List ways to estimate the volume of this rock that may take more than 10 seconds. Be prepared to discuss the accuracy of your method when compared to others' methods.

2. What if instead of a rock, you had a large pile of dirt? Would your ideas above help you find the volume of dirt? Explain.

Suppose your large pile of dirt is as large as a building or a mountain? Surveyors use a tool called a total station to measure the dimensions around the plot of interest (in this case, the sides of the pile).

Figure 3: Total station operation (left), Back-site (right image shown on a yellow tripod), GPS shown on right image as black pole with tablet and round antenna

You are tasked with using a closed traverse and a total station to measure the dimensions of the pile of dirt (shown above and again in figure 2). You walk the traverse and collect data (as shown in figures below).

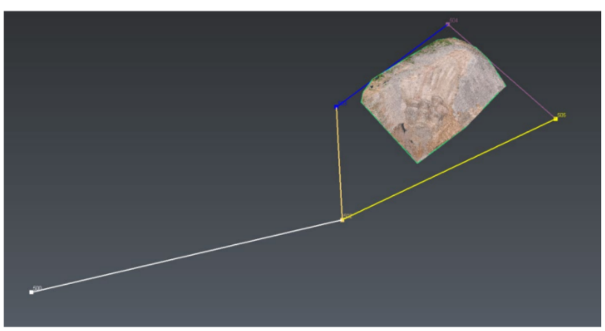
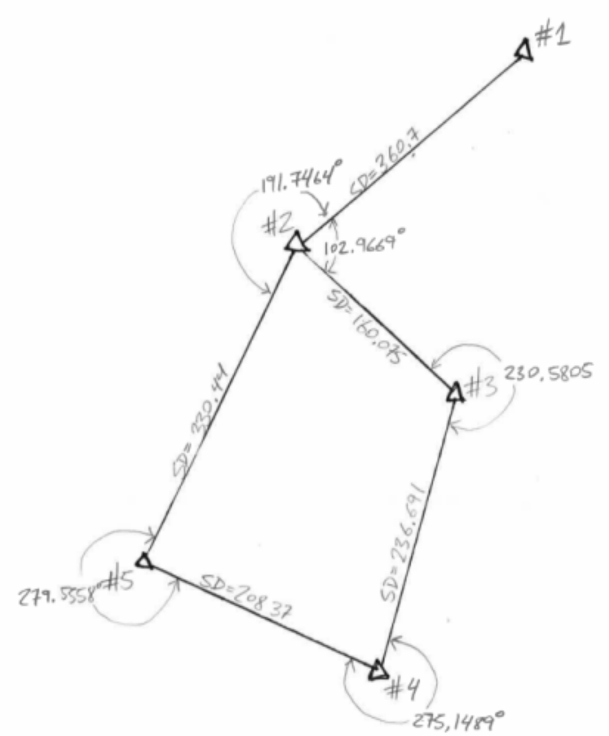
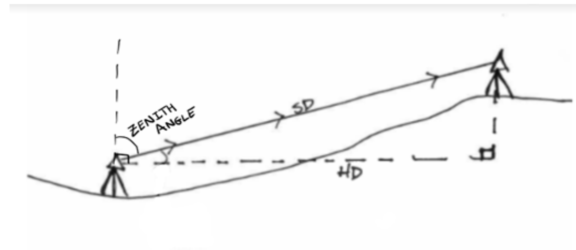


Figure 2: Survey Traverse Path around a Stockpile. The two known points are at the end of the white line, with the three new locations being "turned in" by the total station

Point Number	Description	Horizontal Angle (Decimal Degrees)	Zenith Angle (Decimal Degrees)	Slope Distance
3	NAIL	102.9669444	91.85111111	160.075
4	NAIL	230.5805556	90.21305556	236.691
5	NAIL	275.1488889	88.34666667	208.37
1002	TIE NAIL	279.5558333	90.01722222	330.44
1001	TIE NAIL	191.7463889	89.5675	360.7



When total stations are set up, the ground is often unlevel (we could be surveying on a hilly plot or a steep mountain). Our total station measures the Slope Distance (SD). Look at the diagram at right showing a total station "hitting" a set target.

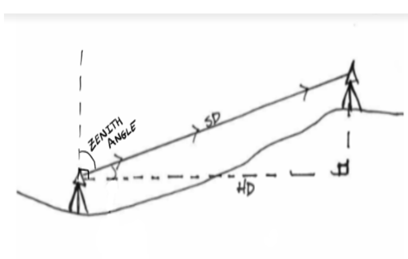


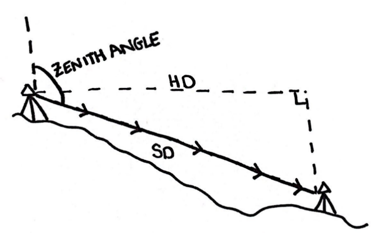
Zenith angles are angle measurements from the vertical and can have values between 0 and 180 degrees. Match the following diagrams with their zenith angle values. Describe the relationships between the total stations in each situation. *Zenith angle choices: 70 degrees, 92 degrees, 115 degrees*

3a. If we are interested in finding the dimensions of the foundation of a large building, which value would we likely use, SD or HD? Explain.

3b. If we are interested in finding the length of a road going up the mountain or hill, which value would we likely use, SD or HD? Explain.

3c. Estimate visually the zenith angles in each diagram below.





3d. What is the relationship between SD, HD, and the Zenith Angle?

4. On your Traverse, you set up the Total Station on a point and shoot the next point at a slope distance of 253.456 ft. Your Zenith Angle for the shot was 115.3213°. What is the Horizontal Distance of this shot?

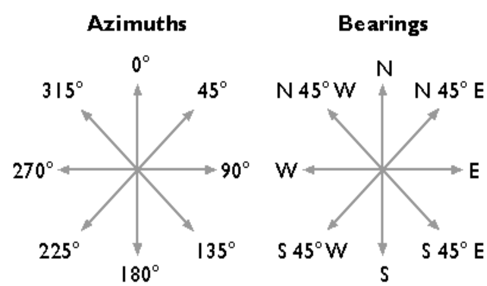
5. Sketch a diagram of the 5-sided traverse loop described below.

Point Number	Interior angle
101	95.1245°
102	108.5431°
103	112.4215°
104	99.7312°
105	123.1586°

6. If your measurements were absolutely perfect, what is the expected sum of the interior angles of a 5-sided traverse loop?

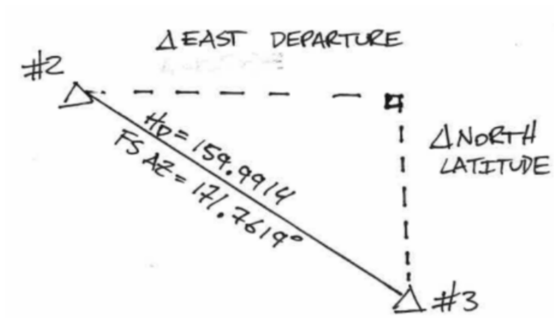
7. Calculate the angular misclosure of the traverse loop described in question 5. Why might this misclosure exist?

Surveyors normally use angles called Azimuths. These are horizontal angles measured from due north rotated in a clockwise direction (as shown to the right).



Traverses have sides that are not horizontal (east-west) and vertical (north-south). We can use information from our total station to find the horizontal and vertical components of the sides of the HD of the traverses.

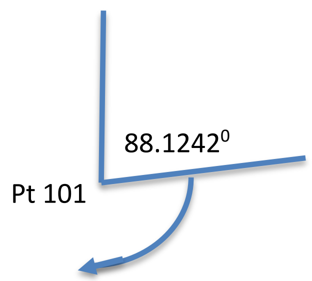
Horizontal components are referred to as Departures and Vertical components are referred to as Latitudes.



- ***Latitude= (HD)* cos(Azimuth)***
- ***Departure= (HD)* sin(Azimuth)***

In task 5 above, there are several different drawings possible due to the fact of not knowing the angle from where the first turned angle is measured from. If you have a starting location and angle to turn from, then only one diagram would have resulted.

8. Using a Backsight Azimuth of 88.1242° as the first angle to turn from, redraw the diagram of the 5-sided traverse loop described in task 5.



9. To find the Latitude and Departure components, we need to first convert Horizontal Angles to Azimuths. Complete the table below by finding the Azimuths angles for each point and the New Backsight Azimuth angles.

Backsight Azimuth	Point Number	Horizontal Angle	Azimuth	New Backsight Azimuth
88.1242	101	95.1245	183.2487	3.2487
3.2487	102	108.5431		
	103	112.4215		
	104	99.7312		
	105	123.1586		

10. Find the Latitude and Departure components for each point in the table below.

Backsight Azimuth	Point Number	Horizontal Distance (HD in feet)	Latitude	Departure
88.1242	101	130		
3.2487	102	110		
	103	125.4		
	104	134.6		
	105	118		

11. Find the sum of the Latitude and the sum of the Departures in the above table. If the 5-point traverse loop is exactly closed, what would you expect the $\sum$(Latitude) and $\sum$(Departures) values would be?

12. What have you created when you find a closed traverse that is accurate to your pile? What additional information is needed to find the volume of the pile?

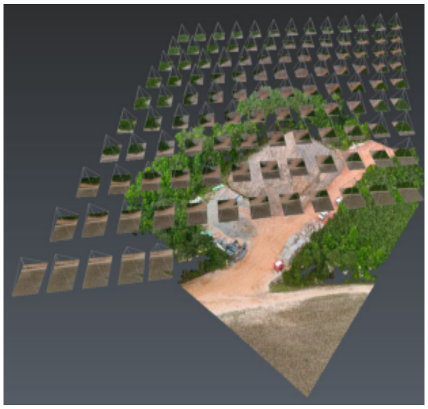
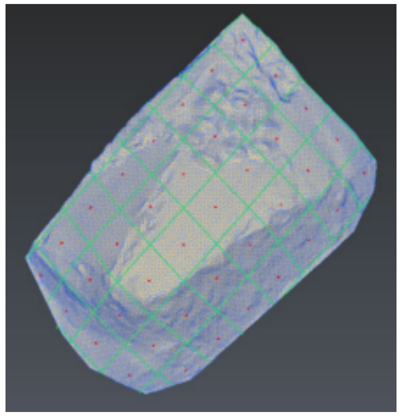
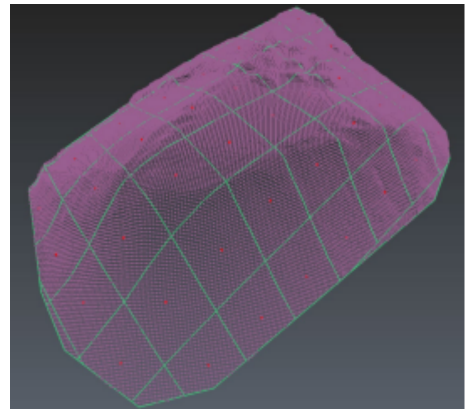
Part 2: Stockpile Volume Calculations

The previous example utilized a total station to establish control points around the base of the stockpile. That same device can be utilized to gather spot locations across the stockpile. This conventional survey method can take a good amount of time to walk across the stockpile and can also present some safety issues if the pile is very steep or has active construction equipment working on or around it.

As technology has improved, we are able to harness the capability of Unmanned Aircraft Systems (UAS) to generate 3D topographic survey information from “old school” photogrammetry principles. When a single point is photographed from multiple angles it can be digitally reconstructed so that its location is represented by 3D points that have an X, Y, and Z location. This methodology can also be viewed as “virtual surveying” since the volumetrics are derived from computer software, but the mathematics are still the same as if they were derived from boots on the ground survey locations. If we have elevations of several points on the top of the stockpile and the ground elevation at the base of the stockpile, we can use the volume formula of a rectangular prism V = LxWxH to find the volume of many small prisms that sum up to the entire pile.

Figure 4: Fixed Wing UAS/Drone (Wingtra) Setup

Figure 5: Photogrammetry Flight

Figure 7: SfM 3D Point Cloud & Image Locations	Figure 8: Virtual Survey 25ft Fishnet Grid (Red Dots)	Figure 9: Virtual Survey 1ft Fishnet Grid (Purple)
		

We will use 2 methods to estimate volume:

I. Accumulate values determined when multiplying area of a 25’x25’ square by the center point height of the square (see figure 8)
II. Accumulate values determined when multiplying area of a 0.7’x0.7’’ square by their center point height of the square (see figure 9)

13. Prior to calculating the volume of the dirt, predict the variability between the stockpile volumes calculated using the variety of survey points available to perform the measurement. Will the results be close to each other, or vary by 25% or more?

Data collected by UAS - Use this for questions 14 and 15…SfM actual pile volume is given in the excel document

14. Method 1: Determine the stockpile volume in cubic feet (above 384ft in elevation) using 39 survey points that were gathered every 25 ft across this dirt stock pile.

15. Method 2: Apply the formula derived to solve the first problem to the data points gathered every 0.7 ft, and compare this volume to Method 1, and the SfM (actual) pile volume.

Vocabulary List:

<table>
<tr><td>Angular Misclosure</td><td>The difference between the measured angles' sum on a traverse and the angle condition for the traverse configuration</td></tr>
<tr><td>Azimuth</td><td>horizontal angles that are measured from the reference meridian in the clockwise direction.

used in compass surveying, plane surveying, where it is generally measured from the north.

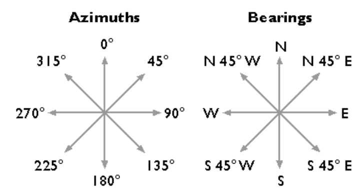
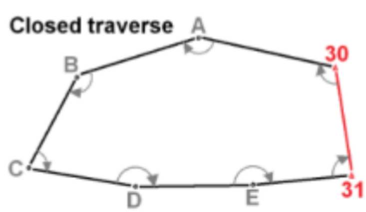
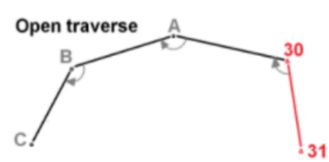

</td></tr>
<tr><td>Misclosure</td><td>the situation where the last in a series of linked traverse lines fails to join up exactly with the first</td></tr>
<tr><td>Total station</td><td>Surveyors use a tool called a total station to measure dimensions – a variety of information is returned by the total station including lengths and angle measures


Figure 3: Total station operation (left), Back-site (right image shown on a yellow tripod), GPS shown on right image as black pole with tablet and round antenna</td></tr>
<tr><td>Ratio of precision</td><td>represents how many feet you would have to traverse with that accuracy to achieve 1 unit of error</td></tr>
<tr><td>Traverse</td><td>Traverse is a method in the field of surveying to establish control networks. It is also used in geodesy. Traverse networks involve placing survey stations along a line or path of travel, and then using the previously surveyed points as a base for observing the next point.

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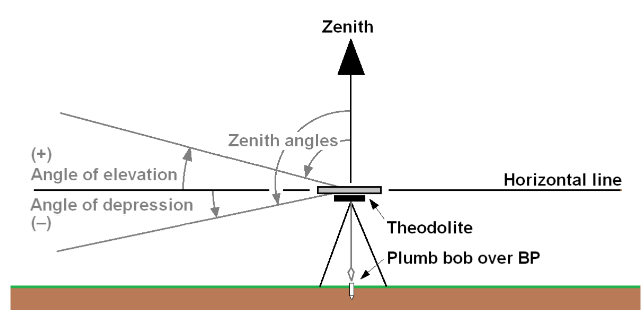
<tr><td>Zenith Angle</td><td>the angle measured from directly above (the zenith) – so perfectly horizontal will have a zenith angle of 90°.
If you are looking above horizontal,the zenith angle will be less than 90°.
If you are looking below horizontal,the zenith angle will be greater than 90°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